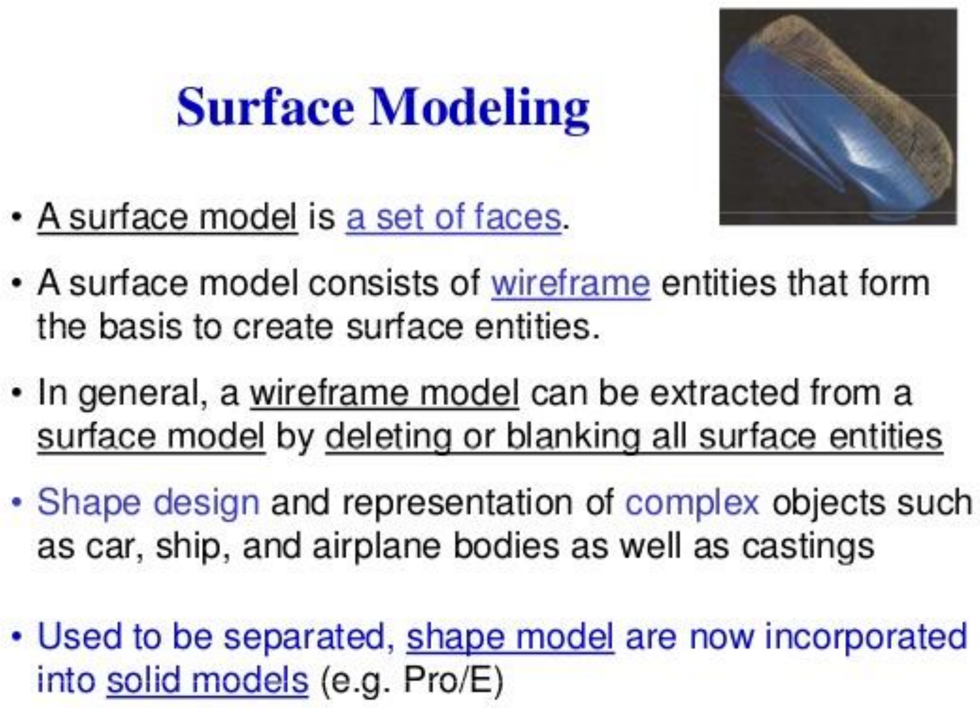
Surface Modeling
Unit 4
CAD

Surface Modeling - A surface model of an object is a more complete and less ambiguous (easily understandable) representation that its wireframe model. Surface models take the modeling of an object one step beyond wireframe models by providing information on surfaces connecting the object edges.



Shape design and representation of complex objects such as car, ship and airplane bodies as well as castings cannot be achieved utilizing wireframe modeling. In such cases, surface modeling must be utilized to describe objects precisely and accurately.

Surface modeling used in various applications such as

1. Calculating mass properties

2. Checking for interference between mating parts

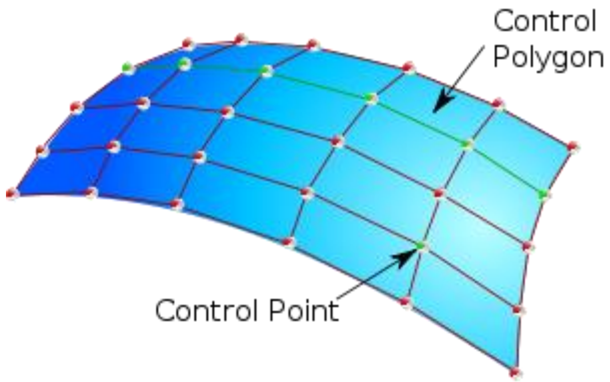
3. Generating cross-sectioned views

4. Generating finite element meshes

5. Generating NC tool paths for continuous path machining

Surface Representation

From CAD/CAM point of view surfaces are as important as curves and solids. We need to have an idea of curves for surface creation. In the same way surfaces form the boundaries of the solids.



Surface Representation is just an extension of representation of curves. We can represent a surface as a series of grid points inside its bounding curves. Surfaces can be in two-dimensional space (planar) or in three-dimensional space (general surfaces). Surface can be described using non-parametric or parametric equations. Surfaces can be represented by equations to pass through all the data points (fitting) or have patches of them connected at the data points (approximations).

There are two types of surfaces.

Analytic surfaces - are based on wireframe entities, and include the plane surface, ruled surface, surface of revolution, and tabulated cylinder.

Synthetic surfaces - are formed from a given set of data points or curves and include the bicubic, Bezier, B-spline, and Coons patches.

Analytic surfaces

1. Plane surface

This is the simplest surface. It requires three non-coincident points to define an infinite plane. The plane surface can be used to generate cross-sectional views by intersecting a surface model with it, generate cross sections for mass property calculations, or other similar applications where a plane is needed. Figure 1 shows a plane surface.

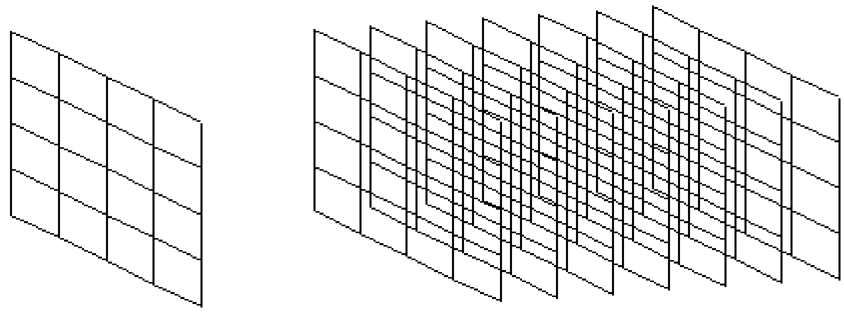
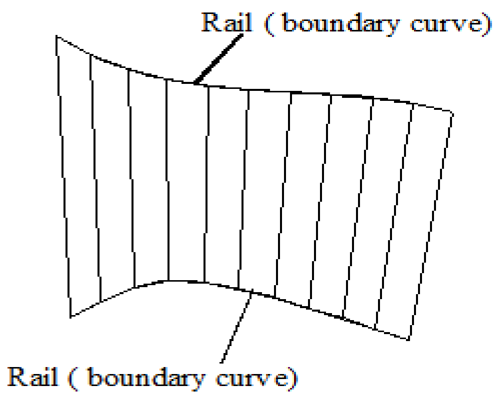


Figure 1. Plane surface

2. Ruled (lofted) surface

A ruled surface can be described as the set of points swept by a moving straight line

This is a linear surface. As shown in Figure, a ruled surface interpolates linearly between two boundary curves that define the surface (rails). Rails can be any wireframe entity.



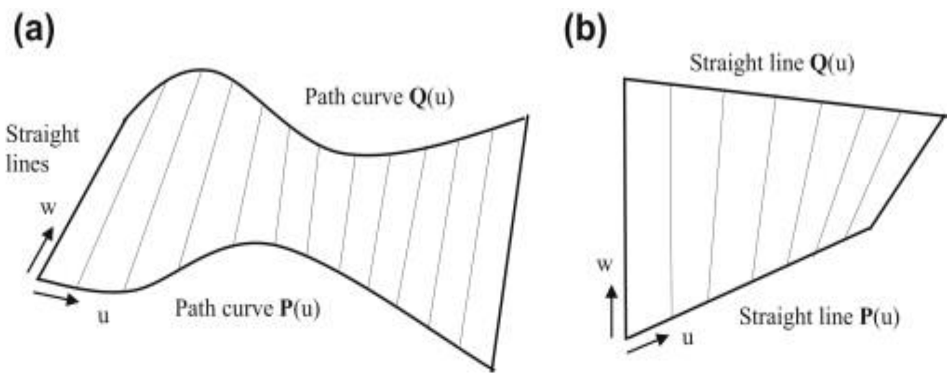


Figure 2. Ruled surface

3. Surface of revolution

This is an axisymmetric surface that can model axisymmetric objects. It is generated by rotating a planar wireframe entity in space about the axis of symmetry at a certain angle

A **surface of revolution** is a **surface** in Euclidean space created by rotating a curve (the generatrix) around an axis of rotation. Examples of **surfaces of revolution** generated by a straight line are cylindrical and conical **surfaces** depending on whether or not the line is parallel to the axis

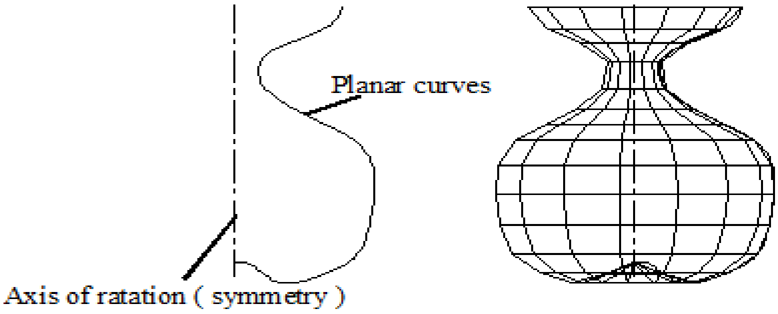


Figure Surface of revolution

4. Tabulated cylinder

This is a surface generated by translating a planar curve at a certain distance along a specified direction (axis of the cylinder) as shown in Figure 4.

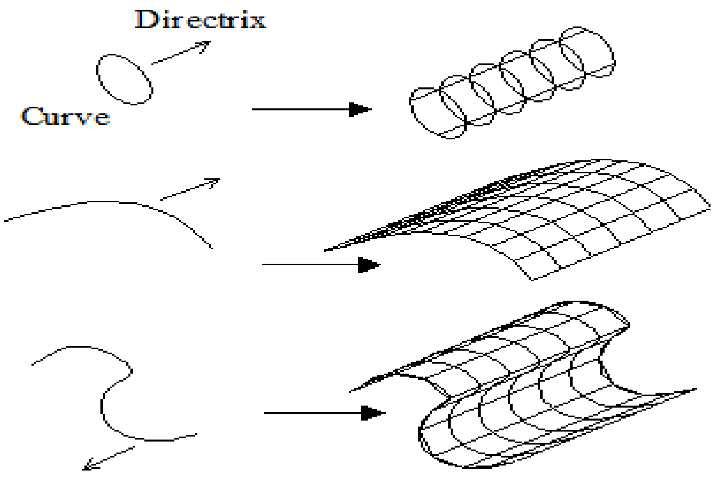
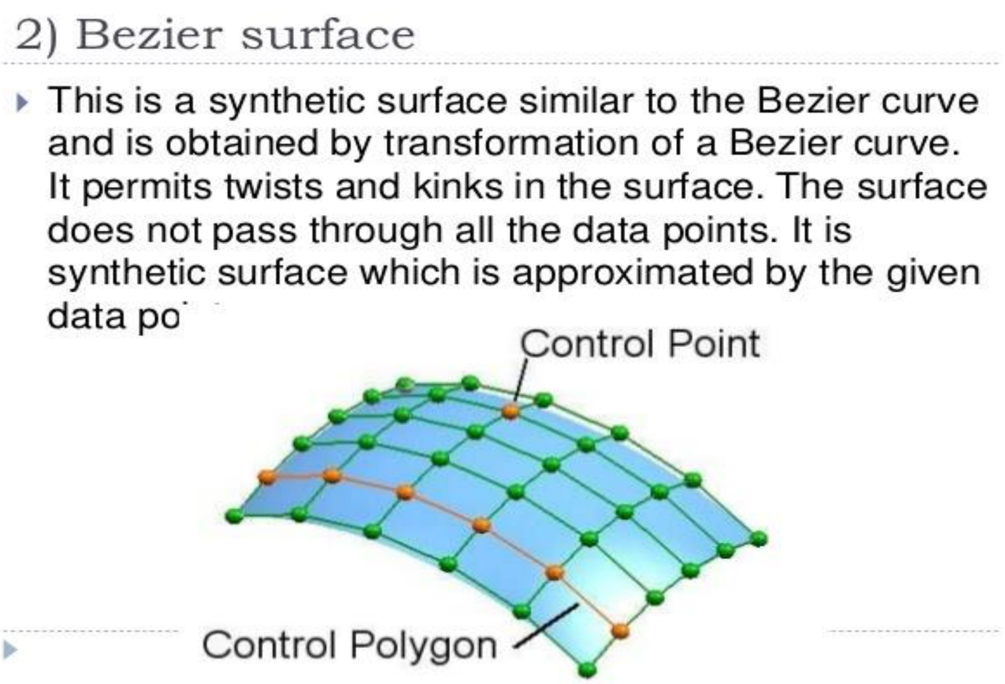


Figure 4. Tabulated cylinder

Synthetic surfaces

1. Bezier surface –

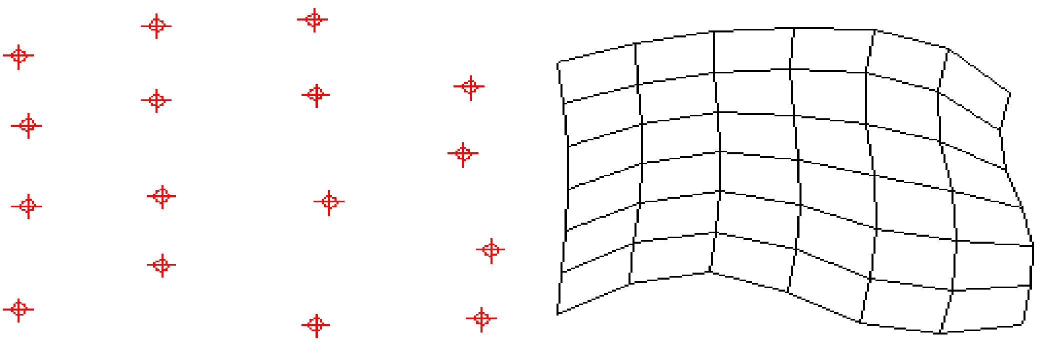
It is a type of synthetic Surface. This is a surface that approximates given input data points (Figure) i.e it does not pass through all given data points. It has global control characteristic like Bezier curve.



2. B-spline surface –

It is a type of synthetic surface

This is a surface that can approximate or interpolate given input data (Figure 6). It is a general surface like the Bezier surface but with the advantage of permitting local control of the surface.

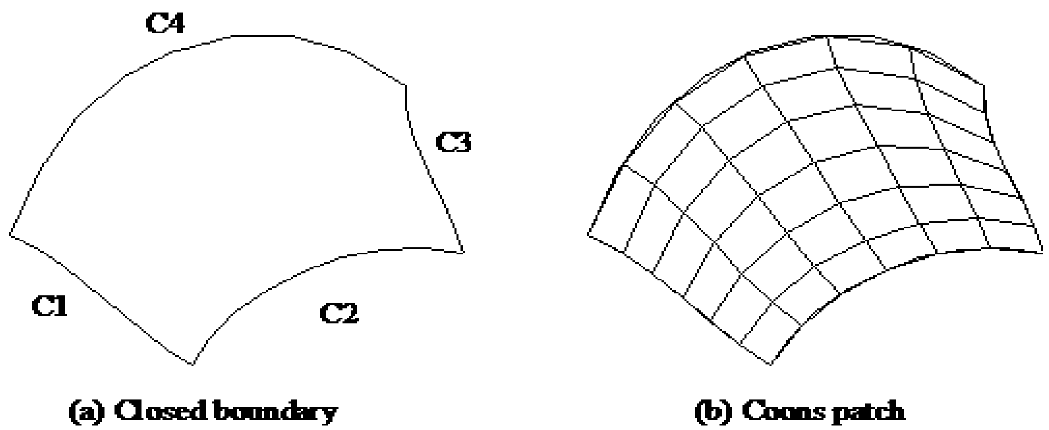


(a) Data points **(b) B-spline**

Figure 6. B-spline surface.

3. Coons patch

The above surfaces are used with either open boundaries or given data points. The Coons patch is used to create a surface using curves that form closed boundaries (Figure 7).



4. Hermite Bicubic Surface

Because the blending functions are linear, the bilinear surface tends to be flat. Bicubic surfaces provide designers with better surface design tools when designing surfaces. The bicubic equation can be written as

$$\mathbf{P}(u,v)=\sum_{i=0}^{3}\sum_{j=0}^{3}C_{ij}u^{i}v^{j},\quad 0\le u\le 1,\ 0\le v\le 1 \qquad (5)$$

$$\mathbf{P}(u,v)=\mathbf{U}^{T}[\mathbf{C}]\mathbf{V},\quad 0\le u\le 1,\ 0\le v\le 1 \qquad (6)$$

where $\mathbf{U}=[u^3\quad u^2\quad u\quad 1]^T, \mathbf{V}=[v^3\quad v^2\quad v\quad 1]^T,$ and the coefficient matrix [C] is

$$[\mathbf{C}]=\begin{bmatrix}\mathbf{C}_{33} & \mathbf{C}_{32} & \mathbf{C}_{31} & \mathbf{C}_{30}\\ \mathbf{C}_{23} & \mathbf{C}_{22} & \mathbf{C}_{21} & \mathbf{C}_{20}\\ \mathbf{C}_{13} & \mathbf{C}_{12} & \mathbf{C}_{11} & \mathbf{C}_{10}\\ \mathbf{C}_{03} & \mathbf{C}_{02} & \mathbf{C}_{01} & \mathbf{C}_{00}\end{bmatrix} \qquad (7)$$

Hermite Bicubic Surface is an extension of Hermite cubic spline. As shown in Figure 2, 16 boundary conditions are required to find the coefficients of the equation. They are the 4 corner data points, the 8 tangent vectors at the corner points (two at each point in the *u* and *v* directions), and the 4 twist vectors at the corner points.

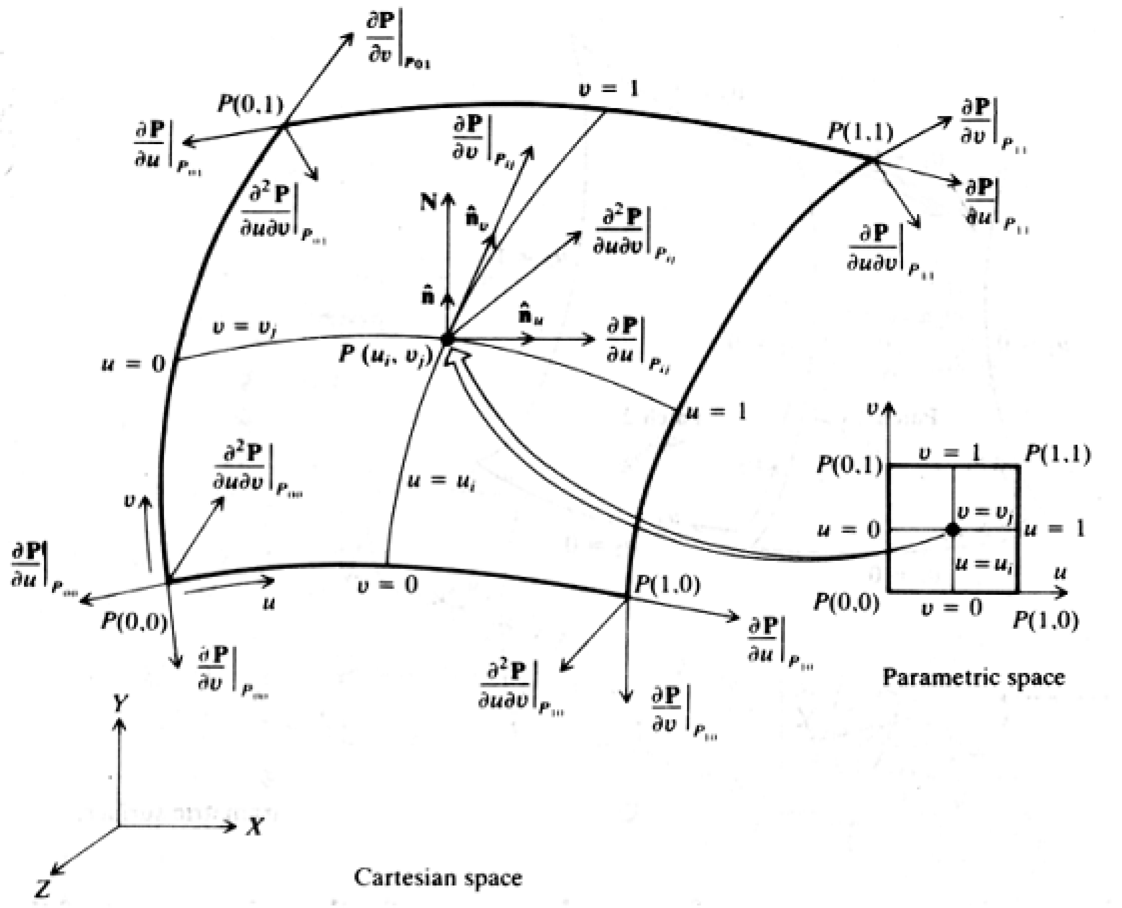


Figure 2. A parametric surface patch with its boundary conditions